Use of Helmholtz Frequency to Determine Volume

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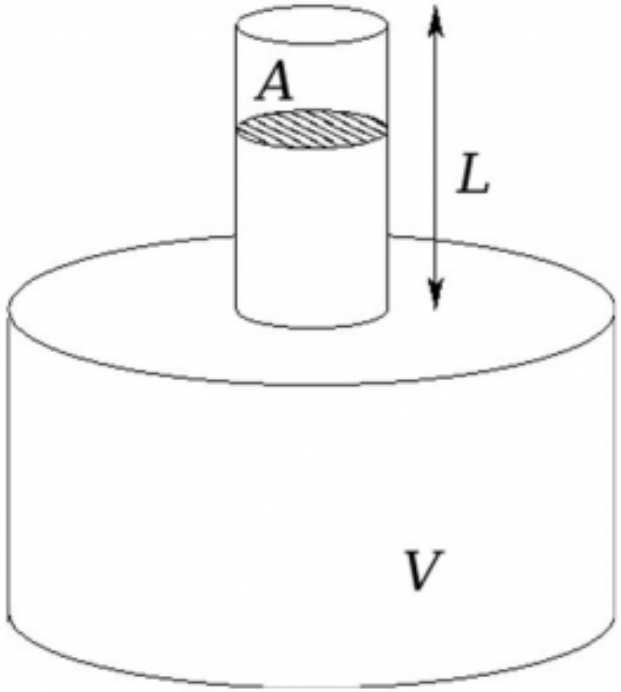
Helmholtz Resonance:

- **A Plug of Air Vibrating on a Pillow of Air**
- **The Same Physics as a Mass on a Spring**

$$F = \frac{c}{2\pi} * \sqrt{\frac{A}{(L + .8 * \sqrt{A}) * V}}$$

- f = Frequency
- c = Speed of Sound in Air
- L = Length of Neck
- A = Cross-Sectional Area of Neck
- V = Volume of Chamber



Questions Asked

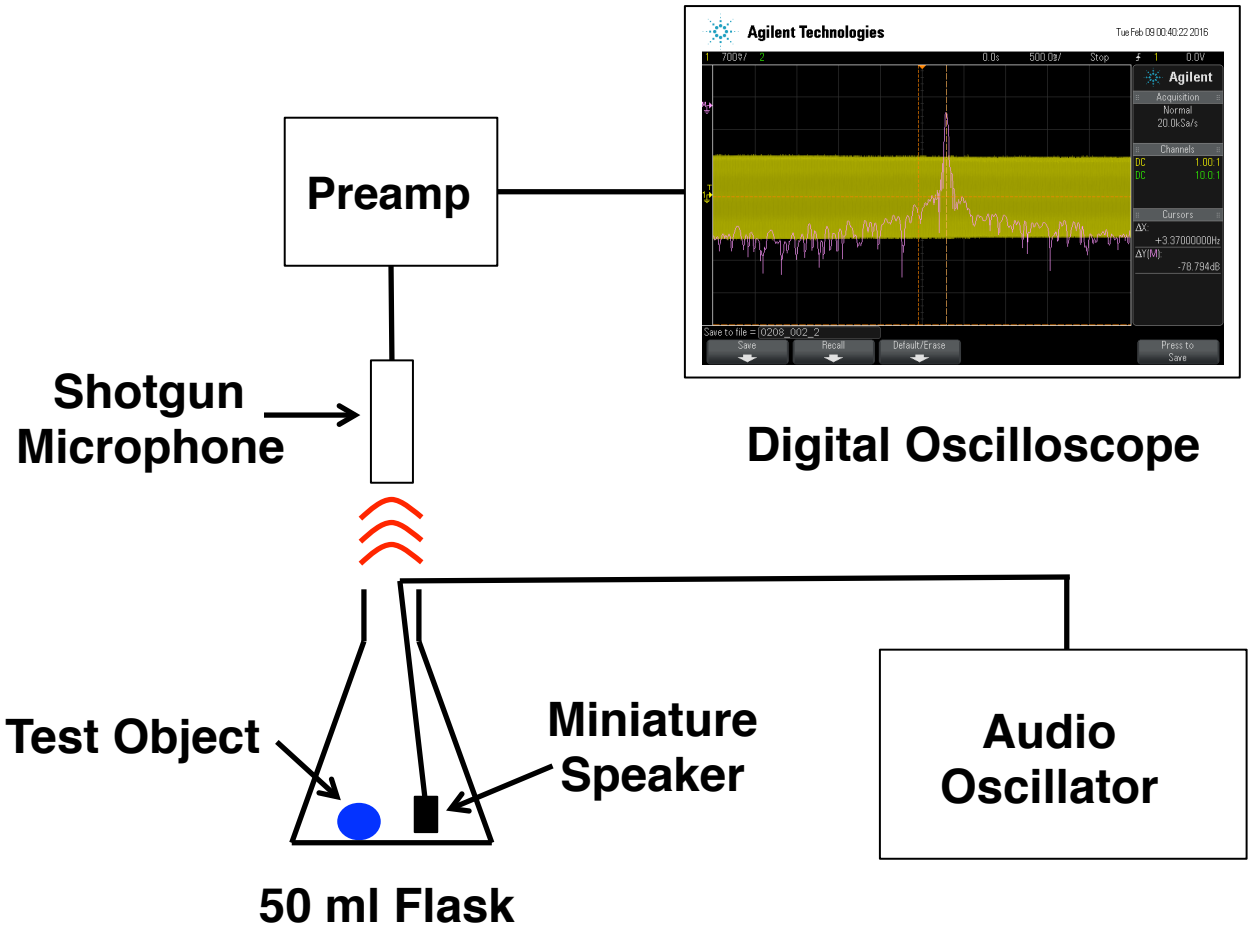
1) Is it possible to build a Helmholtz resonator and characterize its resonant frequency with a precision of 0.01 Hz?

2) Can the resonate frequency be raised by placing objects of known volume into the cavity?

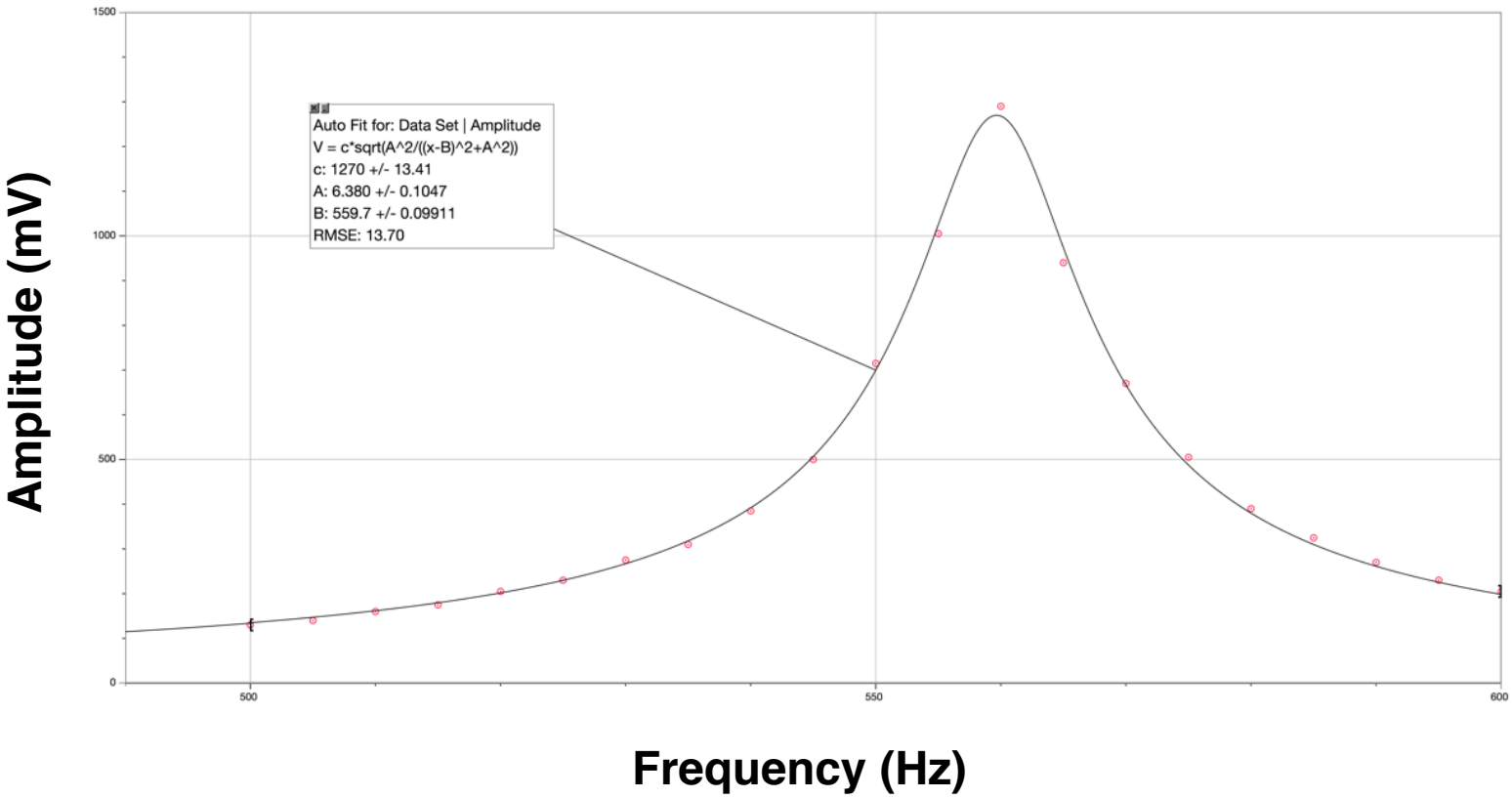
3) Can the volume of an irregularly shaped object therefore be determined by its shift in frequency?

4) Can density be determined with sufficient accuracy to differentiate common gold alloys?

Experimental Apparatus



Helmholtz Resonance

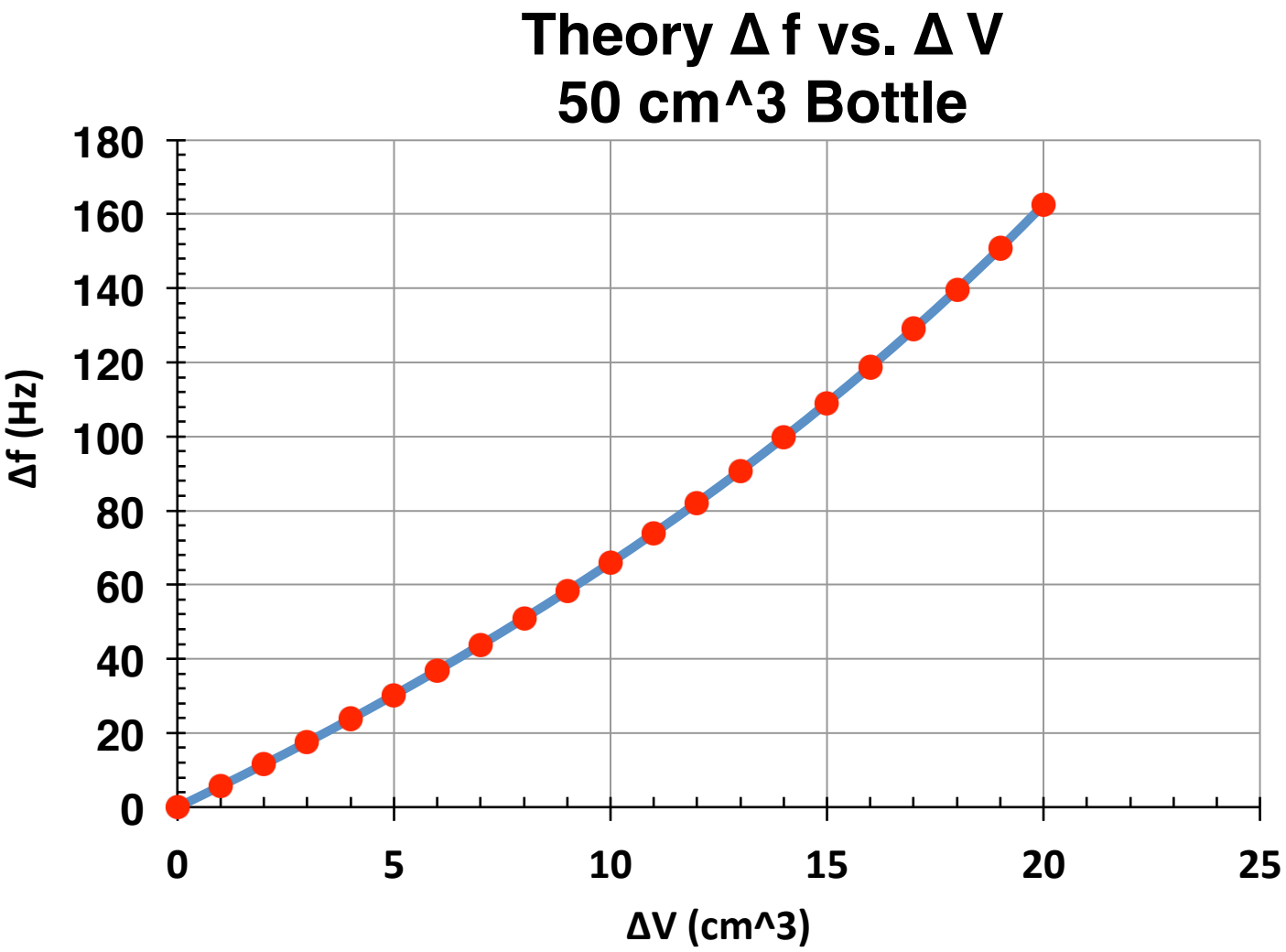


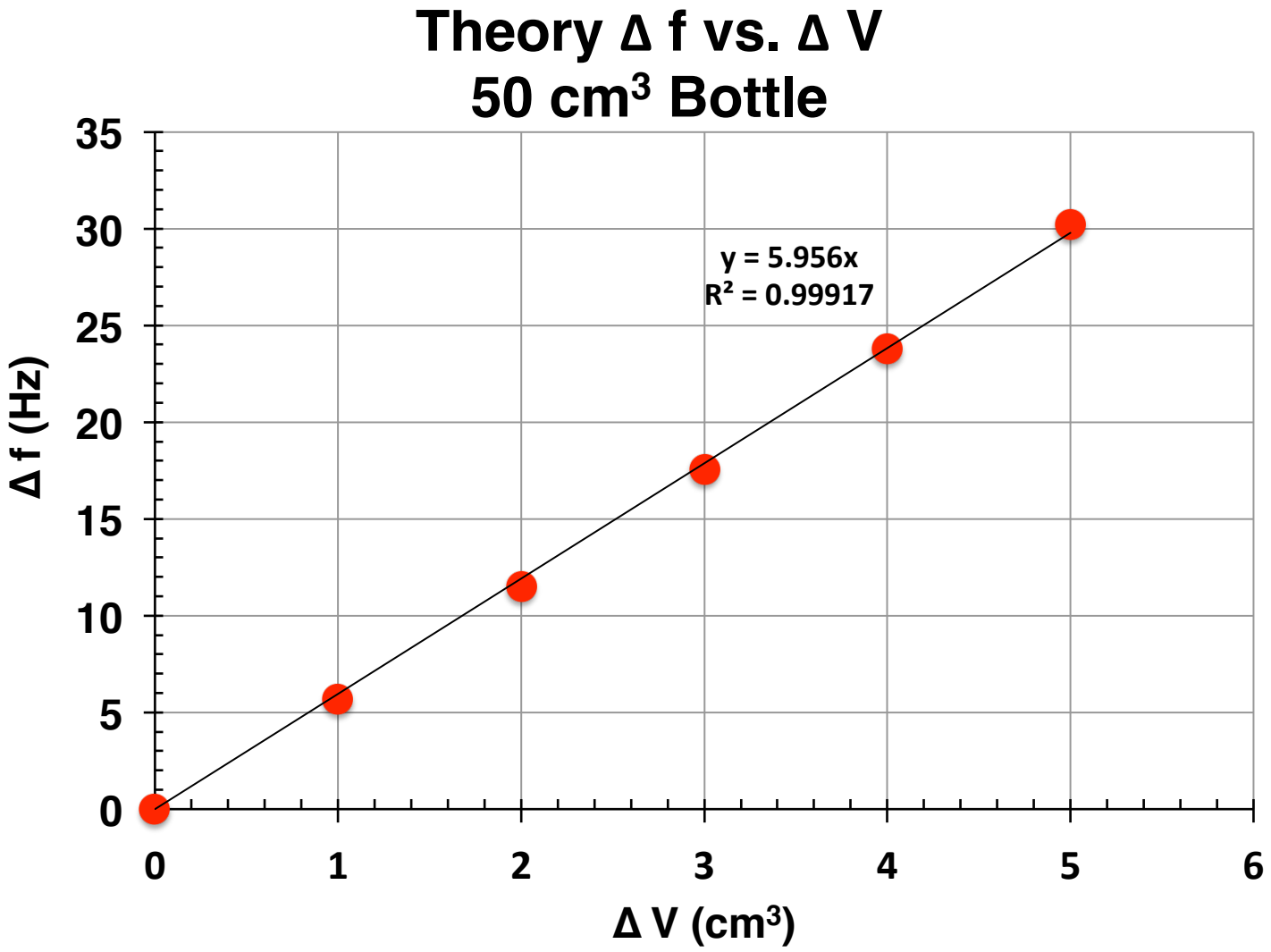
Q-Factor: Parameter describing damping of system

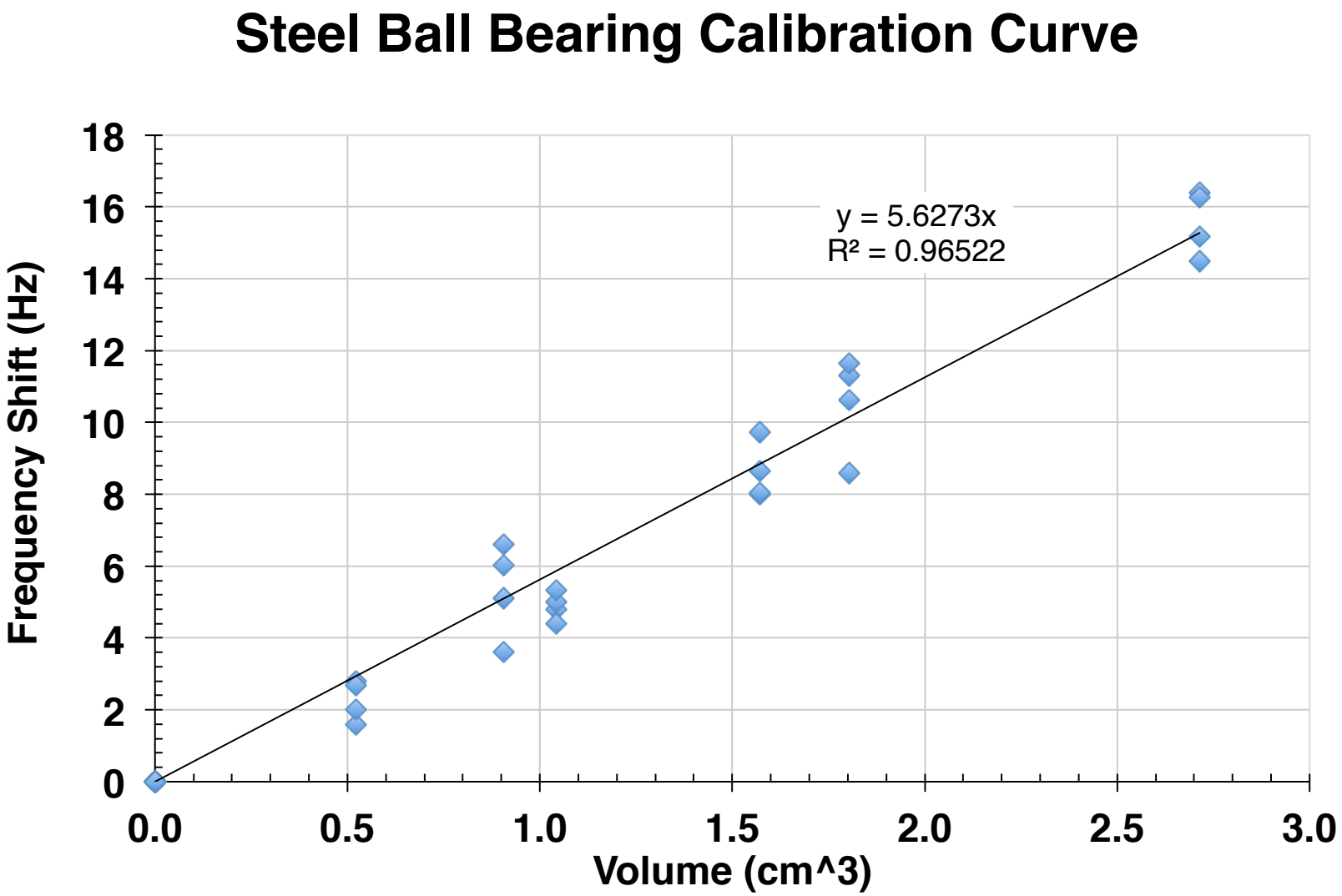
$Q = (f_{peak})/(\Delta f_{halfpower}) = (559.7)/(20) = 27.99$

Empirical Expectations

- **Lump All Constants Together**
- **$f_0 = C / V^{1/2}$**
- **Measure f_0 = 559 Hz for 50 ml Flask**
- **C = 3950 s^{-1} $cm^{3/2}$**
- **Use C to calculate theoretical dependence of the change in volume on the change in frequency.**
- **If the change in volume is small, i.e. < 10%, the change in resonant frequency is linear.**


Theory Δ f vs. Δ V
50 cm^3 Bottle
Δf (Hz)
180
160
140
120
100
80
60
40
20
0
0
5
10
15
20
25
ΔV (cm^3)


Theory Δ f vs. Δ V
50 cm3 Bottle
y = 5.956x
R² = 0.99917
Δ f (Hz)
Δ V (cm3)
0
5
10
15
20
25
30
35
0
1
2
3
4
5
6


Steel Ball Bearing Calibration Curve
y = 5.6273x
R² = 0.96522
Frequency Shift (Hz)
Volume (cm^3)
0
2
4
6
8
10
12
14
16
18
0.0
0.5
1.0
1.5
2.0
2.5
3.0

How Reproducibly Can I Manually Determine The Resonance Frequency?

Trial #	Frequency (Hz)
1	560.68
2	560.08
3	559.29
4	559.21
5	559.66
6	560.02
7	560.06
8	560.36
9	559.08
10	560.31

Standard Deviation: **+/- 0.57 Hz**

Standard Deviation Volume: **+/- 0.10 cm^3**

Measurement of Sample Objects

Wooden Dowell

Volume (cm^3)	Acoustic Volume (cm^3)	Percent Error
1.83	1.54 (+/- 0.11)	-16%
3.12	2.82 (+/- 0.25)	-9.62%
4.36	4.02 (+/- 0.14)	-7.8%

Wire (Folded)

Volume (cm^3)	Acoustic Volume (cm^3)	Percent Error
0.22	0.24 (+/- 0.03)	+9.1%
0.44	0.49 (+/- 0.12)	+11.4%
0.67	0.63 (+/- 0.07)	-6%

Wire (Coiled)

Volume (cm^3)	Acoustic Volume (cm^3)	Percent Error
0.22	0.22 (+/- 0.13)	0%
0.44	0.43 (+/- 0.05)	-2.3%
0.67	0.67 (+/- 0.1)	0%

Unknown Metal Necklace: Is it Gold?

14 Carat Gold Gold Chain Experiment

Place Chain in Flask

Measure $\Delta f = 6.62$ Hz

Calibration: $\Delta f = 5.63$ Hz/cm^3

Calculate Volume = 1.18 cm^3

Measure Mass = 16.7 g

Calculate Density = 14.2 g/cm^3

Calculation verifies 14 Carat count stamped on chain clasp.

Metal	Density
Gold	19.3
Silver	10.5
Platinum	21.4
Palladium	12.0
Copper	9.0
9ct	10.9 to 12.7
14ct	12.9 to 14.6
18ct Yellow	15.2 to 15.9
18ct White	14.7 to 16.9
22ct	17.7 to 17.8
Sterling Silver	10.2 to 10.3
950 Platinum	20.1

Conclusions and Future Directions

- This method can be used to calculate the volume of irregularly shaped objects with ca. 10% error.
- This technique was sufficiently accurate to enable the differentiation of common gold alloys.
- This method is desirable where volume and density determination by water displacement is impractical.
- Automating the process of finding the resonant frequency would increase reproducibility and accuracy.